17/1/2022

B. Sc. Part I (Hons.)

2nd Paper

INTEGRAL CALCULUS

Reduction Formula

Q. If $I_n = \int \cos^n x \, dx$, find a relation between I_n and I_{n-2}.

Soln. $\because I_n = \int \cos^n x \, dx = \int \cos^{n-1} x \cos x \, dx$

Now, integrating by parts, we get-

$$\Rightarrow I_n = \cos^{n-1} x \int \cos x \, dx - \int \left[\frac{d}{dx} \left(\cos^{n-1} x \right) \int \cos x \, dx \right] dx$$

$$\Rightarrow I_n = \cos^{n-1} x \sin x + \int (n-1) \cos^{n-2} x \cdot \sin x \cdot \sin x \, dx$$

$$\Rightarrow I_n = \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \, dx$$

$$\Rightarrow I_n = \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x \left(1 - \cos^2 x \right) dx$$

$$= \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x \, dx - (n-1) \int \cos^n x \, dx$$

$$\Rightarrow I_n = \cos^{n-1} x \sin x + (n-1) I_{n-2} - (n-1) I_n$$

$$\Rightarrow I_n\left[1+(n-1)\right] = (n-1)\, I_{n-2} + \cos^{n-1}x \sin x$$

$$\Rightarrow n I_n = (n-1)\, I_{n-2} + \cos^{n-1}x \sin x.$$

This is the required relationship.

Q. If $I_n = \int \sin^n x\, dx$, establish a relationship between I_n and I_{n-2}.

Soln. $\because$ $I_n = \int \sin^n x\, dx = \int \sin^{n-1}x \cdot \sin x\, dx$

$$\Rightarrow I_n = \sin^{n-1}x \int \sin x\, dx - \int\left[\frac{d}{dx}\left(\sin^{n-1}x\right)\int \sin x\, dx\right]dx$$

$$\Rightarrow I_n = -\sin^{n-1}x \cdot \cos x + \int (n-1)\sin^{n-2}x \cos^2 x\, dx$$

$$\Rightarrow I_n = -\sin^{n-1}x \cdot \cos x + (n-1)\int \sin^{n-2}x\,(1-\sin^2 x)\, dx$$

$$\Rightarrow I_n = -\sin^{n-1}x \cos x + (n-1)\int \sin^{n-2}x - (n-1)\int \sin^n x\, dx$$

$$\Rightarrow I_n = -\sin^{n-1}x \cos x + (n-1)\, I_{n-2} - (n-1)\, I_n$$

$$\Rightarrow I_n(1+n-1) = (n-1)\, I_{n-2} - \sin^{n-1}x \cos x$$

$$\Rightarrow n I_n = (n-1)\, I_{n-2} - \sin^{n-1}x \cos x.$$

This is the required relationship.